Extending the Real Business Cycle Model Part 2

Felix Wellschmied

UC3M

Macroeconomics II

Outlook

- We will now end the analysis of the RBC model with two more exercises. Both serve to better bridge the gap to the New Keynesian model.
- First, we will introduce money in the economy. You will see that this changes little (or nothing).
- Second, we will introduce imperfect competition in the economy. Imperfect competition on its own will only change the steady state of the economy. Howerver, once we make competition time-varying, business cycle dynamics will arise.

Money

The idea

- So far, we have considered the real economy, where only quantities are determined.
- We are now going to introduce money. You will see that this changes little about real variables in the RBC model because prices are flexible.
- For money to really matter for real variables, we will require some price stickiness which will be the New Keynesian model.

Why money exists

- Money is an asset that
  - is a store of value.
  - is a medium of exchange.
  - is a unit of account.
- The most common justification given for why we use money is that it is easier than barter.
- To capture this idea, we ware going to look at two theories:
  - money in the utility function.
  - a cash in advance constraint.

The process of money

- Households have to pay now a price P_t in terms of money for real quantities.
- We will assume that the quantity of money is set exogenously (central bank).
- We will assume that money growth follows a stochastic AR(1) process. You can think about the central bank sometimes printing more and sometimes printing less than normal.
- We allow for a constant average money supply growth rate. You will see that this will give us inflation.

$$\ln M_{t+1} - \ln M_t = (1 - \rho_m) m^{ss} + \rho_m (\ln M_t - \ln M_{t-1}) + \epsilon_t^m. \qquad (1)$$

Rewriting the process

As the level of the money supply is changing over time, the process is not stationary. It will turn out that real money balances, M_{t+1}/P_t, are stationary. Hence, we rewrite the process in terms of real money balances:

$$\ln M_{t+1} - \ln M_t = (1-\rho_m)m^{ss} + \rho_m(\ln M_t - \ln M_{t-1}) + \epsilon_t^m \tag{2}$$

$$\ln M_{t+1} - \ln P_t + \ln P_t - \ln P_{t-1} - \ln M_t + \ln P_{t-1} = \\ (1-\rho_m)m^{ss} + \rho_m(\ln M_t - \ln P_{t-1} + \ln P_{t-1} - \ln P_{t-2} - \ln M_{t-1} + \ln P_{t-2}) + \epsilon_t^m \tag{3}$$

$$\Delta \ln m_{t+1} + \pi_t = (1-\rho_m)m^{ss} + \rho_m(\Delta \ln m_t + \pi_{t-1}) + \epsilon_t^m, \tag{4}$$

where $m_t = M_{t+1}/P_t$ and π_t is the inflation rate.

Households

- Households receive factor payments in nominal terms.
- Households trade a nominal bond with each other that pays a pre-determined nominal interest of i_{t-1}. We will see that this is equivalent to firms issuing these bonds.
- They enter the period with a stock of money M_t and decide that period about the stock of money to carry over to the next period M_{t+1}.
- They pay real lump-sum taxes T_t. You will see that we need these.

The budget constraint with money

$$P_tC_t + P_tK_{t+1} + B_{t+1} + M_{t+1} - M_t$$
$$= W_tH_t + R_tK_t + (1 + i_{t-1})B_t + P_t\Pi_t - P_tT_t + (1 - \delta)K_tP_t. \quad (5)$$

Note, W_t and R_t are now the nominal variables. Let us rewrite the constrained in real terms:

$$C_t + K_{t+1} + \frac{B_{t+1}}{P_t} + \frac{M_{t+1} - M_t}{P_t}$$
$$= \frac{W_t}{P_t}H_t + \frac{R_t}{P_t}K_t + (1 + i_{t-1})\frac{B_t}{P_t} + \Pi_t - T_t + (1 - \delta)K_t. \quad (6)$$

Money in the utility function

- We require some mechanism why people wish to hold money instead of bonds.

- We start by assuming that households derive utility from holding real money balances:

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \phi \frac{H_t^{1+\eta}}{1+\eta} + \varphi \ln\left(\frac{M_{t+1}}{P_t}\right). \tag{7}$$

- M_{t+1} are the today determined money balances carried over to the next period.

- The idea is that real money balances allow households to avoid barter. The more money they hold, the less time consuming is shopping.

- This is a very crude way to model money demand as the demand does not depend on the level of consumption.

The household problem

$$\max_{C_t, K_{t+1}, H_t, B_{t+1}, M_{t+1}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \phi \frac{H_t^{1+\eta}}{1+\eta} + \varphi \ln \left(\frac{M_{t+1}}{P_t} \right) \right) \right\} \quad (8)$$

s.t.

$$C_t + K_{t+1} + \frac{B_{t+1}}{P_t} + \frac{M_{t+1} - M_t}{P_t} = \frac{W_t}{P_t} H_t + \frac{R_t}{P_t} K_t + (1 + i_{t-1}) \frac{B_t}{P_t} + \Pi_t - T_t + (1-\delta) K_t. \quad (9)$$

$$\ln M_{t+1} - \ln M_t = (1 - \rho_m) m^{ss} + \rho_m (\ln M_t - \ln M_{t-1}) + \epsilon_t^m. \quad (10)$$

First order conditions

$$\Lambda_t = \mathbb{E}_0 \Big\{ \sum_{t=0}^{\infty} \beta^t \Big[\frac{C_t^{1-\gamma}}{1-\gamma} - \phi \frac{H_t^{1+\eta}}{1+\eta} + \varphi \ln\left(\frac{M_{t+1}}{P_t}\right) - \lambda_t \Big(C_t + K_{t+1} + \frac{B_{t+1}}{P_t} + \frac{M_{t+1} - M_t}{P_t} - \frac{W_t}{P_t} H_t - \frac{R_t}{P_t} K_t - (1 + i_{t-1}) \frac{B_t}{P_t} - \Pi_t + T_t - (1-\delta) K_t \Big) \Big] \Big\}. \quad (11)$$

$$\frac{\partial \Lambda_t}{\partial H_t} : \phi H_t^{\eta} = \lambda_t \frac{W_t}{P_t} \quad (12)$$

$$\frac{\partial \Lambda_t}{\partial C_t} : C_t^{-\gamma} = \lambda_t \quad (13)$$

$$\frac{\partial \Lambda_t}{\partial K_{t+1}} : \beta^t \lambda_t = \mathbb{E}_t \Big\{ \beta^{t+1} \lambda_{t+1} \left(\frac{R_{t+1}}{P_{t+1}} + (1-\delta) \right) \Big\} \quad (14)$$

First order conditions II

$$\Lambda_t = \mathbb{E}_0\Big\{\sum_{t=0}^{\infty}\beta^t\Big[\frac{C_t^{1-\gamma}}{1-\gamma} - \phi\frac{H_t^{1+\eta}}{1+\eta} + \varphi \ln\left(\frac{M_{t+1}}{P_t}\right) - \lambda_t\Big(C_t + K_{t+1} + \frac{B_{t+1}}{P_t} + \frac{M_{t+1}-M_t}{P_t} - \frac{W_t}{P_t}H_t - \frac{R_t}{P_t}K_t - (1+i_{t-1})\frac{B_t}{P_t} - \Pi_t + T_t - (1-\delta)K_t\Big)\Big]\Big\}. \quad (15)$$

$$\frac{\partial \Lambda_t}{\partial B_{t+1}} : \beta^t\lambda_t = \mathbb{E}_t\Big\{\beta^{t+1}\lambda_{t+1}(1+i_t)\frac{P_t}{P_{t+1}}\Big\} \quad (16)$$

$$\frac{\partial \Lambda_t}{\partial M_{t+1}} : \beta^t\frac{\varphi}{P_t}\left(\frac{M_{t+1}}{P_t}\right)^{-1} + \beta^{t+1}\mathbb{E}_t\frac{\lambda_{t+1}}{P_{t+1}} = \beta^t\frac{\lambda_t}{P_t} \quad (17)$$

Combining terms

Hours optimality:

$$\phi H_t^{\eta} = C_t^{-\gamma} \frac{W_t}{P_t} \tag{18}$$

Euler equation:

$$C_t^{-\gamma} = \beta \mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} \left(\frac{R_{t+1}}{P_{t+1}} + (1-\delta) \right) \Big\} \tag{19}$$

Bond optimality:

$$C_t^{-\gamma} = \beta \mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} (1+i_t) \frac{P_t}{P_{t+1}} \Big\} \tag{20}$$

Money optimality:

$$C_t^{-\gamma} = \varphi \left(\frac{M_{t+1}}{P_t} \right)^{-1} + \beta \mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \Big\} \tag{21}$$

Preview of the New Keynesian mechanism

- One key ingredient for output fluctuations in the New Keynesian model will be shocks to the nominal interest rate and price expectations.

- The intuition for this can be seen already in this model using the bond equation.

- However, we will also see that the mechanism is not operational in this model because of fully flexible prices.

Log-linearizing bond optimality

To understand the mechanism, let us assume for the moment that $i^{ss} = i = \frac{1}{\beta} - 1$. Then log-linearizing yields:

$$C_t^{-\gamma} = \beta(1 + i_t)\mathbb{E}_t \left\{ C_{t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right\} \tag{22}$$

$$(1 - \gamma \hat{C}_t) = (1 - \gamma \hat{C}_{t+1} + \hat{P}_t - \mathbb{E}_t \hat{P}_{t+1})[1 + (1-\beta)\hat{i}_t] \tag{23}$$

$$\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t = \frac{1}{\gamma} \left[\hat{P}_t - \mathbb{E}_t \hat{P}_{t+1} + (1-\beta)\hat{i}_t \right] \tag{24}$$

Expected consumption growth is high when the cost of consumption today is high relative to the expected cost of consumption tomorrow. Note, prices show up in terms of the expected change of the price level (expected inflation).

Consumption growth is high when the nominal interest rate is high.

Preview of the New Keynesian mechanism

$$\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t = \frac{1}{\gamma}\left[\hat{P}_t - \mathbb{E}_t \hat{P}_{t+1} + (1-\beta)\hat{i}_t\right] \tag{25}$$

These insights will be key when introducing a central bank that can affect interest rates and future price expectations. If the central bank raises the expected costs of consumption tomorrow, it will create consumption today! Similarly, if it decreases the interest rate, it will increase consumption today.

The Fisher equation

However, in our model, consumption growth will be independent of changes in prices or the nominal interest rate. To see this, combine the Euler equation and the bond equation:

$$C_t^{-\gamma} = \beta \mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} \left(\frac{R_{t+1}}{P_{t+1}} + (1-\delta) \right) \Big\} \tag{26}$$

$$C_t^{-\gamma} = \beta \mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} (1+i_t) \frac{P_t}{P_{t+1}} \Big\}. \tag{27}$$

Hence,

$$\mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} \left(\frac{R_{t+1}}{P_{t+1}} + (1-\delta) \right) \Big\} = \mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} \frac{1+i_t}{1+\pi_{t+1}} \Big\}. \tag{28}$$

The Fisher equation II

$$\mathbb{E}_t\Big\{C_{t+1}^{-\gamma}\left(\frac{R_{t+1}}{P_{t+1}}+(1-\delta)\right)\Big\}=\mathbb{E}_t\Big\{C_{t+1}^{-\gamma}\frac{1+i_t}{1+\pi_{t+1}}\Big\}. \tag{29}$$

This is called the Fisher equation. It states that the real return on capital is linked to the nominal return on bonds and inflation. When inflation is high, nominal interest rates need to be high. Hence, with flexible prices, all changes in the nominal interest rate are offset by changes in the inflation rate leading to no independent fluctuations in either. To break this logic, i.e., have fluctuations in the real interest rate, we require price stickiness.

Comparing money and bond optimality

$$C_t^{-\gamma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\gamma} (1+i_t) \frac{P_t}{P_{t+1}} \right\} \tag{30}$$

$$C_t^{-\gamma} = \varphi \left(\frac{M_{t+1}}{P_t} \right)^{-1} + \beta \mathbb{E}_t \left\{ C_{t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right\}. \tag{31}$$

Combining the two equations yields:

$$C_t^{-\gamma} = \varphi \left(\frac{M_{t+1}}{P_t} \right)^{-1} + \frac{C_t^{-\gamma}}{(1+i_t)} \tag{32}$$

$$\varphi \left(\frac{M_{t+1}}{P_t} \right)^{-1} = C_t^{-\gamma} \left[1 - \frac{1}{1+i_t} \right] \tag{33}$$

Note, bonds, other than money, pay an interest rate. Households hold money until the marginal utility of holding money compensates them for the forgone interest rate.

Money demand equations

We can solve now for the demand of real money balances:

$$C_t^{-\gamma}\left[1-\frac{1}{1+i_t}\right]=\varphi\left(\frac{M_{t+1}}{P_t}\right)^{-1} \tag{34}$$

$$\varphi\left(\frac{M_{t+1}}{P_t}\right)^{-1}=C_t^{-\gamma}\frac{i_t}{1+i_t} \tag{35}$$

$$\frac{M_{t+1}}{P_t}=m_t=\varphi C_t^{\gamma}\left(\frac{1+i_t}{i_t}\right) \tag{36}$$

Real money demand depends positively on the level of consumption and negatively on the nominal interest rate.

Firm behavior

The firm maximizes

$$\max_{K_t, H_t} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{-\gamma}}{C_0^{-\gamma}} \left[P_t K_t^{\alpha} \left(A_t H_t \right)^{1-\alpha} - W_t H_t - R_t K_t \right] \right\} \tag{37}$$

with

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1} \tag{38}$$

Firm optimality

The first order conditions are given by

$$\frac{R_t}{P_t} = \alpha K_t^{\alpha-1} (A_t H_t)^{1-\alpha} \tag{39}$$

$$\frac{W_t}{P_t} = (1-\alpha) K_t^{\alpha} A_t^{1-\alpha} H_t^{-\alpha} \tag{40}$$

which is exactly the same as before. Moreover, as firms operate under perfect competition, profits are zero, $\Pi_t = 0$.

Equilibrium

We are now going to derive several properties of the equilibrium with money:

1. We will derive the long run-inflation rate.
2. We derive the equilibrium in the bonds market.
3. We derive the equilibrium level of taxes.
4. We use these properties to simplify the household's budget constraint.

Steady state inflation

Given the money demand equation:

$$\frac{M_{t+1}}{P_t} = \varphi C_t^{\gamma} \left(\frac{1 + i_t}{i_t} \right), \tag{41}$$

we can directly compute the steady state inflation rate. In steady state, C_t and i_t are constant. Hence, real money balances, $\frac{M_{t+1}}{P_t}$, must be constant. This is only possible when the growth rate of the price level equals the growth rate of the money supply: $\pi^{ss} = m^{ss}$.

Bond market equilibrium

- The household trades bonds only with itself.
- As a result, $B_t = 0$ in all t.
- Remember, this is also an equilibrium when bonds are issued by firms.

Equilibrium taxation

We will assume that government spending is zero and that the budget balances each period by adjusting the lump-sum tax. The government earns revenues by printing money with which it could buy real resources. Hence, the budget constraint reads

$$\frac{M_{t+1} - M_t}{P_t} + T_t = 0 \tag{42}$$

from which directly follows that

$$T_t = -\frac{M_{t+1} - M_t}{P_t}, \tag{43}$$

i.e., the government distributes its seigniorage revenues back to the household.

Equilibrium budget constraint

Using the bond market equilibrium and equilibrium taxation, the household's budget constraint simplifies to

$$C_t + K_{t+1} = \frac{W_t}{P_t} H_t + \frac{R_t}{P_t} K_t + \Pi_t + (1 - \delta) K_t. \tag{44}$$

Finally, using the firm optimality we have

$$C_t + K_{t+1} = Y_t + (1 - \delta) K_t, \tag{45}$$

i.e., all output is either consumed or invested.

Equilibrium

- We can divide the economy into a real side and a nominal side.
- The real side determines quantities and real prices.
- The nominal side determines the price level and nominal variables.
- This implies that money has no effect on real quantities.
- This is called the classical dichotomy.

Equilibrium, the real side

Given the states A_t and K_t, the following set of equations determine all real variables over time.

$$Y_t = K_t^{\alpha} \left(A_t H_t\right)^{1-\alpha} \tag{46}$$

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t \tag{47}$$

$$\frac{R_t}{P_t} = \alpha K_t^{\alpha-1} \left(A_t H_t\right)^{1-\alpha} \tag{48}$$

$$\frac{W_t}{P_t} = (1-\alpha) K_t^{\alpha} A_t^{1-\alpha} H_t^{-\alpha} \tag{49}$$

$$\phi H_t^{\eta} = C_t^{-\gamma} \frac{W_t}{P_t} \tag{50}$$

$$C_t^{-\gamma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\gamma} \left(\frac{R_{t+1}}{P_{t+1}} + (1-\delta) \right) \right\} \tag{51}$$

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1} \tag{52}$$

Equilibrium, the nominal side

Take the nominal interest rate as given (more on this in a second), the price level adjusts such that real money supply facilitates the level of consumption:

$$m_t = \varphi C_t^{\gamma} \left(\frac{1 + i_t}{i_t} \right). \tag{53}$$

Given $\Delta \ln m_{t+1}$, the money growth rule determines the resulting inflation:

$$\Delta \ln m_{t+1} + \pi_t = (1 - \rho_m) m^{ss} + \rho_m (\Delta \ln m_t + \pi_{t-1}) + \epsilon_t^m. \tag{54}$$

Finally, the Fisher equation determines the nominal interest rate:

$$\mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} \left(\frac{R_{t+1}}{P_{t+1}} + (1 - \delta) \right) \Big\} = \mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} \frac{1 + i_t}{1 + \pi_{t+1}} \Big\}. \tag{55}$$

Calibration

- We are going to use the same calibration as before with an inverse labor supply elasticity $\eta = 0.5$.
- We will look at the non-inflationary steady-state, $m^{ss} = 0$.
- For the moment, I am interest in qualitative results. Hence, I simply set $\rho_m = 0$, i.e., money is a random walk. I match the HP-filtered standard deviation of CPI inflation (0.006) with $\sigma_m = 0.0022$.
- The parameter of the utility of money, φ, matter for the level of money demand which is unimportant for its dynamics. We will set $\varphi = 1$.

Qualitative assessment

- We have seen that the real side of the economy is unchanged.
- Hence, we are only going to consider what happens to nominal values.
- We start by asking what happens after a positive productivity shock.
- Afterward, we consider an increase in the money growth rate.

Productivity shocks and the nominal side

We know that after an increase in productivity, real consumption increases. Consider the money demand equation:

$$m_t = \varphi C_t^{\gamma} \left(\frac{1 + i_t}{i_t} \right). \tag{56}$$

To facilitate the consumption increase, real money balances, m_t, need to increase. Because M_t is given, prices decrease, i.e., π_t needs to be negative.

Productivity shocks and the nominal side II

Next, consider the Fisher equation:

$$\mathbb{E}_t\Big\{C_{t+1}^{-\gamma}\left(\frac{R_{t+1}}{P_{t+1}}+(1-\delta)\right)\Big\}=\mathbb{E}_t\Big\{C_{t+1}^{-\gamma}\frac{1+i_t}{1+\pi_{t+1}}\Big\}. \tag{57}$$

We know that the real interest rate increases after an increase in productivity. Moreover, as consumption is increasing over time, π_{t+1} must be decreasing. Hence, i_t, in general, can either increase or decrease. It turns out, with a random walk in money, the nominal interest rate does not move at all. One can see that from combining (16) and (17). First, we can write the latter as:

$$\frac{\lambda_t}{P_t}=\frac{\varphi}{M_{t+1}}+\beta\mathbb{E}_t\frac{\lambda_{t+1}}{P_{t+1}} \tag{58}$$

$$\frac{\lambda_t}{P_t}=\sum_{s=0}^{\infty}\beta^s\frac{\varphi}{M_{t+1+s}} \tag{59}$$

Productivity shocks and the nominal side III

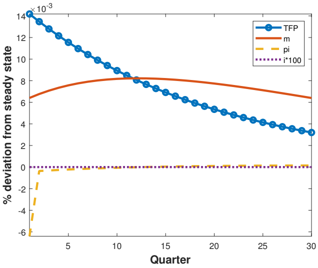
$$\frac{\lambda_t}{P_t} = \sum_{s=0}^{\infty} \beta^s \frac{\varphi}{M_{t+1+s}} \tag{60}$$

However, M_{t+1+s} is exogenous and, hence, does not respond to the shock. Therefore, $\frac{\lambda_t}{P_t}$ and $\frac{\lambda_{t+1}}{P_{t+1}}$ must also remain constant. However, from (16):

$$\frac{\lambda_t}{P_t} = \mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1}}{P_{t+1}} (1 + i_t) \right\} \tag{61}$$

it follows that i_t must remain constant.

Impulse response functions



- Remember that money is neutral for real variables.
- Hence, the model implies that inflation is countercyclical, $CORR(Y_t, \pi_t) = -0.47$.
- This is counterfactual. In the data, inflation is procyclical, $CORR(Y_t, \pi_t) = 0.32$.
- We will see how the New Keynesian model fixes this implication. Key will be that output will increase in response to an inflationary shock, i.e., money will no longer be neutral.

Money shocks and the nominal side

Now consider an increase in the money supply growth rate. The money supply growth is

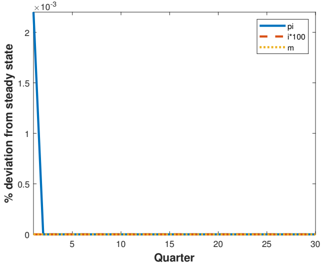
$$\Delta \ln m_{t+1} + \pi_t = (1 - \rho_m) m^{ss} + \rho_m (\Delta \ln m_t + \pi_{t-1}) + \epsilon_t^m \tag{62}$$

One can show that i_t is again a constant. As consumption is unchanged in all periods, and

$$m_t = \varphi C_t^{\gamma} \left(\frac{1 + i_t}{i_t} \right), \tag{63}$$

it must be that real money is unchanged in all periods, $\Delta \ln m_{t+1} = 0$. Hence, $\pi_t = \epsilon_t^m$. As money growth is a random walk and money growth fully adjusts in the first period, it directly follows that $\pi_{t+1} = 0$, i.e., inflation is very short-lived.

Impulse response functions



- Money neither affects real quantities nor the real money supply.
- Resulting from the random walk in the money supply, inflation lasts only one period.
- Changes in money affect only the price level as in the quantity theory of money.

The long run optimal inflation rate

Though the money supply does not affect real quantities, it can affect welfare. To see this, consider the money demand function

$$m_t = \varphi C_t^{\gamma} \frac{1 + i_t}{i_t}. \tag{64}$$

Higher inflation does not affect C_t but it increases i_t which decreases m_t.

In our model, a higher m_t makes people happy by making their lives easier, hence, we want to decrease i_t. The lowest feasible interest rate is $i_t = 0$ when the marginal utility of holding money becomes zero:

$$\varphi \left(\frac{M_{t+1}}{P_t} \right)^{-1} = C_t^{-\gamma} \left[1 - \frac{1}{1 + i_t} \right]. \tag{65}$$

The long run optimal inflation rate II

Now consider the Fisher equation in steady state with $i^{ss} = 0$:

$$\left(\frac{R}{P}\right)^{ss} + (1-\delta) = \frac{1}{1+\pi^{ss}} \tag{66}$$

$$\pi^{ss} = \frac{1}{\left(\frac{R}{P}\right)^{ss} + (1-\delta)} - 1 \leq 0, \tag{67}$$

i.e., the optimum inflation in the long run is negative. This is called the Friedman rule after Milton Friedman who has obtained the Nobel price for his research on monetary economics. The intuition is: The nominal interest rate is a tax on holding money. Setting the nominal interest rate to zero avoids this distortionary tax. Note, Europe has had (involuntarily) since 2009 a monetary policy almost consistent with the Friedman rule.

Cash in advance constraint

- This far, we have rationalized money holdings by households deriving explicit utility from it.
- We are now going to consider an alternative foundation: Households need to hold money to make nominal purchases.
- That is, households have to satisfy an additional constraint:

$$M_t \geq P_t C_t$$

The household problem

$$\max_{C_t, K_{t+1}, H_t, B_{t+1}, M_{t+1}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \phi \frac{H_t^{1+\eta}}{1+\eta} \right) \right\} \tag{68}$$

s.t.

$$C_t + K_{t+1} + \frac{B_{t+1}}{P_t} + \frac{M_{t+1} - M_t}{P_t} = \frac{W_t}{P_t} H_t + \frac{R_t}{P_t} K_t + (1 + i_{t-1}) \frac{B_t}{P_t} + \Pi_t - T_t + (1-\delta) K_t. \tag{69}$$

$$\frac{M_t}{P_t} \geq C_t \tag{70}$$

$$\ln M_{t+1} - \ln M_t = (1-\rho_m) m^{ss} + \rho_m (\ln M_t - \ln M_{t-1}) + \epsilon_t^m. \tag{71}$$

First order conditions

$$\Lambda_t = \mathbb{E}_0\Big\{\sum_{t=0}^{\infty}\beta^t\Big[\frac{C_t^{1-\gamma}}{1-\gamma} - \phi\frac{H_t^{1+\eta}}{1+\eta}$$
$$- \lambda_t\Big(C_t + K_{t+1} + \frac{B_{t+1}}{P_t} + \frac{M_{t+1}-M_t}{P_t} - \frac{W_t}{P_t}H_t - \frac{R_t}{P_t}K_t$$
$$- (1+i_{t-1})\frac{B_t}{P_t} - \Pi_t + T_t - (1-\delta)K_t\Big) - \mu_t\Big(C_t - \frac{M_t}{P_t}\Big)\Big]\Big\}. \quad (72)$$

$$\frac{\partial\Lambda_t}{\partial H_t} : \phi H_t^{\eta} = \lambda_t\frac{W_t}{P_t} \quad (73)$$
$$\frac{\partial\Lambda_t}{\partial C_t} : C_t^{-\gamma} - \mu_t = \lambda_t \quad (74)$$
$$\frac{\partial\Lambda_t}{\partial K_{t+1}} : \beta^t\lambda_t = \beta^{t+1}\mathbb{E}_t\Big\{\lambda_{t+1}\Big(\frac{R_{t+1}}{P_{t+1}} + (1-\delta)\Big)\Big\} \quad (75)$$

First order conditions II

$$\Lambda_t = \mathbb{E}_0\Big\{\sum_{t=0}^{\infty}\beta^t\Big[\frac{C_t^{1-\gamma}}{1-\gamma} - \phi\frac{H_t^{1+\eta}}{1+\eta}$$
$$-\lambda_t\Big(C_t + K_{t+1} + \frac{B_{t+1}}{P_t} + \frac{M_{t+1}-M_t}{P_t} - \frac{W_t}{P_t}H_t - \frac{R_t}{P_t}K_t$$
$$-(1+i_{t-1})\frac{B_t}{P_t} - \Pi_t + T_t - (1-\delta)K_t\Big) - \mu_t\Big(C_t - \frac{M_t}{P_t}\Big)\Big]\Big\}. \quad (76)$$

$$\frac{\partial\Lambda_t}{\partial B_{t+1}} : \beta^t\lambda_t = \beta^{t+1}\mathbb{E}_t\Big\{\lambda_{t+1}(1+i_t)\frac{P_t}{P_{t+1}}\Big\} \quad (77)$$
$$\frac{\partial\Lambda_t}{\partial M_{t+1}} : \beta^t\frac{\lambda_t}{P_t} = \beta^{t+1}\mathbb{E}_t\left(\frac{\lambda_{t+1}}{P_{t+1}} + \frac{\mu_{t+1}}{P_{t+1}}\right) \quad (78)$$

Optimal consumption

Optimal consumption:

$$C_t^{-\gamma} = \lambda_t + \mu_t \tag{79}$$

The marginal utility of consumption needs to be equal the shadow price of having one less unit in the budget constrained and tightening the cash in advance constrained by one unit.

The Euler equation

The Euler equation becomes:

$$C_t^{-\gamma} - \mu_t = \beta \mathbb{E}_t \left\{ [C_{t+1}^{-\gamma} - \mu_{t+1}] \left(\frac{R_{t+1}}{P_{t+1}} + (1 - \delta) \right) \right\} \tag{80}$$

The marginal benefit of consuming today (the MUC minus the costs of tightening the cash constraint) must equal the expected marginal benefits of saving (the expectations over the MUC tomorrow minus the tightening of the constraint tomorrow times the returns on savings).

Money optimality

$$\frac{C_t^{-\gamma} - \mu_t}{P_t} = \beta \mathbb{E}_t \left(\frac{C_{t+1}^{-\gamma} - \mu_{t+1}}{P_{t+1}} + \frac{\mu_{t+1}}{P_{t+1}} \right) \tag{81}$$

$$C_t^{-\gamma} = \mu_t + \beta \mathbb{E}_t \frac{C_{t+1}^{-\gamma}}{1 + \pi_{t+1}}. \tag{82}$$

By forgoing one unit of consumption today and investing the resources in money, the household obtains a relaxed cash in advance constrained and one expected real unit of consumption tomorrow. Importantly, it does not earn any returns.

Bond optimality

Finally, lets substitute the Lagrange multiplier in the bond equation:

$$C_t^{-\gamma} - \mu_t = \beta \mathbb{E}_t \left\{ [C_{t+1}^{-\gamma} - \mu_{t+1}] \frac{(1 + i_t)}{1 + \pi_{t+1}} \right\} \tag{83}$$

Properties of the equilibrium

We will derive now that

- in any equilibrium with a positive interest rate on bonds, the constraint on money balances must be binding.

- money is no longer neutral in the short run.

Properties of the constraint

Compare the bond and money optimality:

$$\frac{\lambda_t}{P_t} = \beta \mathbb{E}_t \Big\{ \lambda_{t+1} \frac{(1+i_t)}{P_{t+1}} \Big\} \tag{84}$$

$$\frac{\lambda_t}{P_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{P_{t+1}} + \frac{\mu_{t+1}}{P_{t+1}} \right\}. \tag{85}$$

Suppose the cash in advance constraint is not binding, i.e., $\mu_t = \mu_{t+1} = 0$. In that case, the nominal interest rate must be zero. Put differently, when money has no value, households are only willing to hold it when bonds pay no return. For any equilibrium with a positive interest rate, the constraint must be binding, i.e., $\frac{M_t}{P_t} = m_t = C_t$

Non-neutrality of money

- Note, the Euler equation depends on the multiplier of the cash in advance constraint.
- Hence, it depends on money.
- Hence, other than with money in the utility function, money is no longer neutral.

Non-neutrality of money II

$$C_t^{-\gamma} - \mu_t = \beta \mathbb{E}_t \Big\{ [C_{t+1}^{-\gamma} - \mu_{t+1}] \left(\frac{R_{t+1}}{P_{t+1}} + (1-\delta) \right) \Big\} \tag{86}$$

When the constraint today is particularly binding relative to the expected constraint tomorrow, $\mu_t >> \mathbb{E}_t \mu_{t+1}$, the marginal utility of consumption today needs to be relatively large. That is, the household reduces its consumption today.

Equilibrium, the real side

Except for the Euler equation, this is exactly the same as before:

$$Y_t = K_t^{\alpha} (A_t H_t)^{1-\alpha} \tag{87}$$

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t \tag{88}$$

$$\frac{R_t}{P_t} = \alpha K_t^{\alpha-1} (A_t H_t)^{1-\alpha} \tag{89}$$

$$\frac{W_t}{P_t} = (1-\alpha) K_t^{\alpha} A_t^{1-\alpha} H_t^{-\alpha} \tag{90}$$

$$\phi H_t^{\eta} = C_t^{-\gamma} \frac{W_t}{P_t} \tag{91}$$

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1} \tag{92}$$

$$C_t^{-\gamma} - \mu_t = \beta \mathbb{E}_t \left\{ [C_{t+1}^{-\gamma} - \mu_{t+1}] \left(\frac{R_{t+1}}{P_{t+1}} + (1-\delta) \right) \right\} \tag{93}$$

Equilibrium, the nominal side

Using the cash in advanced constraint, we know real money balances:

$$m_t = C_t. \tag{94}$$

Given $\Delta \ln m_{t+1}$, the money growth rule determines the resulting inflation:

$$\Delta \ln m_{t+1} + \pi_t = (1-\rho_m)m^{ss} + \rho_m(\Delta \ln m_t + \pi_{t-1}) + \epsilon_t^m. \tag{95}$$

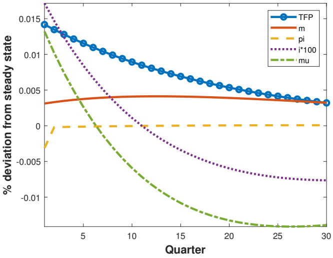
Using the money optimality, we obtain the multiplier:

$$\mu_t = C_t^{-\gamma} - \beta \mathbb{E}_t \frac{C_{t+1}^{-\gamma}}{1+\pi_{t+1}}. \tag{96}$$

Using the bond optimality, we obtain the nominal interest rate:

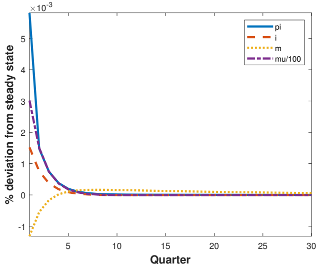
$$C_t^{-\gamma} - \mu_t = \beta \mathbb{E}_t \Big\{ [C_{t+1}^{-\gamma} - \mu_{t+1}] \frac{(1+i_t)}{1+\pi_{t+1}} \Big\} \tag{97}$$

Impulse response functions



- As before, a productivity shock decreases prices and, hence, increases real money balances.
- The increase in consumption makes the cash in advance constraint tighter.
- As the value of money increases, the return on bonds needs to increase.

Impulse response functions II



- The money supply growth shock increases inflation.
- As a result, the nominal interest rate increases and real money balances fall.
- As a result, the cash in advance constraint becomes tighter.
- Recall that $m_t = C_t$, i.e., consumption also falls. As holding money becomes more expensive, workers decide to consume less.

The long run optimal inflation rate

- The household would prefer not to face a binding cash in advance constraint.
- We have seen that this is only consistent with $i^{ss} = 0$.
- Which is again the Friedman rule.

Mark-up shocks

The idea

- So far, we assume perfect competition in the goods market, i.e., $\frac{\partial P}{\partial Y} = 0$.
- We are now going to introduce imperfect competition.
- Imperfect competition reduces the level of output as firms reduce output to increase profits.
- If market power is cyclical, this may introduce business cycle fluctuations.

The set-up

- For simplicity, we will assume labor is the only factor of production.
- Households trade again a nominal bond with each other.
- Otherwise, we will not change anything at the household side.
- It is convenient to split the production side into two entities:
  1. There is a unit mass of intermediate goods producers which produce a differentiable good using labor.
  2. There is a final goods producer which simply bundles these intermediate goods into a final output good.

The household problem

$$\max_{C_t, B_{t+1}, H_t} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \phi \frac{H_t^{1+\eta}}{1+\eta} \right) \right\} \tag{98}$$

s.t.

$$P_t C_t + B_{t+1} = W_t H_t + \Pi_t + (1 + i_{t-1}) B_t, \tag{99}$$

where P_t is the price of the consumption good, Π_t are the profits from the intermediate goods producers, and i_t is the nominal interest rate.

First order conditions

$$\Lambda_t = \mathbb{E}_0\Big\{\sum_{t=0}^{\infty}\beta^t\Big[\frac{C_t^{1-\gamma}}{1-\gamma} - \phi\frac{H_t^{1+\eta}}{1+\eta}$$
$$- \lambda_t[P_tC_t + B_{t+1} - W_tH_t - \Pi_t - (1+i_{t-1})B_t]\Big]\Big\}. \quad (100)$$

$$\frac{\partial\Lambda_t}{\partial C_t} : C_t^{-\gamma} = \lambda_t P_t \quad (101)$$
$$\frac{\partial\Lambda_t}{\partial B_{t+1}} : \beta^t\lambda_t = \beta^{t+1}\mathbb{E}_t\lambda_{t+1}(1+i_t) \quad (102)$$
$$\frac{\partial\Lambda_t}{\partial H_t} : \phi H_t^{\eta} = \lambda_t W_t \quad (103)$$

Household optimality

Combining terms gives us the Euler equation and the optimal hours condition:

$$C_t^{-\gamma} = \beta(1 + i_t)\mathbb{E}_t \left\{ C_{t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right\} \tag{104}$$

$$\phi H_t^{\eta} = C_t^{-\gamma} \frac{W_t}{P_t}. \tag{105}$$

Final goods producer

The final goods producer uses all available intermediate input goods $y_{j,t}$ and bundles them to the (real) final output good using as production technology:

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} \, dj \right)^{\frac{\mu}{\mu-1}} . \tag{106}$$

- μ is the substitution elasticity between intermediate input goods j.
- As intermediate input goods are imperfect substitutes, intermediate goods producers will have market power.
- As $\mu \to \infty$ goods become perfect substitutes (perfect competition).

Optimal input usage

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} dj \right)^{\frac{\mu}{\mu-1}}. \tag{107}$$

The production function is such that aggregate output is highest when all intermediate inputs are used in the same proportion. To see this, assume they are not and $\ln y_{j,t} \sim N(\ln \bar{y}, \sigma_y^2)$. Remember, $\mathbb{E}x^a = \exp(\bar{x})^a \exp(0.5\sigma_x^2)^{a^2}$ when x is log-normally distributed. Hence,

$$Y_t = \left(\bar{y}^{\frac{\mu-1}{\mu}} \exp(0.5\sigma_y^2)^{(\frac{\mu-1}{\mu})^2} \right)^{\frac{\mu}{\mu-1}} \tag{108}$$

$$Y_t = \left(\bar{y} \exp(0.5\sigma_y^2)\right) \exp(0.5\sigma_y^2)^{-1/\mu}. \tag{109}$$

The first term is simply the mean of the distribution (the mean of the log-normal distribution). The second term is decreasing in the variance of the usage of idiosyncratic quantities.

Optimization

The final goods producer takes the intermediate goods prices, $p_{j,t}$, as given and sells the final good at price P_t. It chooses inputs to maximize profits:

$$\max_{y_{j,t}} \left\{ P_t \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} dj \right)^{\frac{\mu}{\mu-1}} - \int_0^1 p_{j,t} y_{j,t} dj \right\}. \tag{110}$$

Optimality implies $\frac{\partial \Pi_t}{\partial y_{j,t}} = 0$:

$$P_t \frac{\mu - 1}{\mu} y_{j,t}^{\frac{\mu-1}{\mu}-1} \frac{\mu}{\mu - 1} \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} dj \right)^{\frac{\mu}{\mu-1}-1} = p_{j,t}. \tag{111}$$

Optimization II

$$P_t y_{j,t}^{\frac{-1}{\mu}} \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} dj \right)^{\frac{1}{\mu-1}} = p_{j,t} \tag{112}$$

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\mu} \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} dj \right)^{\frac{\mu}{\mu-1}} \tag{113}$$

Plugging in the production function yields:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\mu} Y_t. \tag{114}$$

The demand for an intermediate input good depends positively on aggregate output and negatively on the price of the intermediate input good relative to the aggregate price. The slope of the demand curve in logs is $-\mu$.

The price of the final output good

The final goods producer operates under perfect competition and, hence, makes zero profit:

$$\underbrace{P_t Y_t}_{\text{Revenues}} - \underbrace{\int_0^1 p_{j,t} y_{j,t} dj}_{\text{Input costs}} = 0. \tag{115}$$

Plugging in (114) yields:

$$P_t Y_t = \int_0^1 p_{j,t}^{1-\mu} P_t^{\mu} Y_t dj \tag{116}$$

Which yields the aggregate price index:

$$P_t = \left(\int_0^1 p_{j,t}^{1-\mu} dj \right)^{\frac{1}{1-\mu}}. \tag{117}$$

Intermediate goods producers

The intermediate goods producers maximize discounted profits:

$$\pi_{j,t} = p_{j,t} y_{j,t} - W_t h_{j,t}, \tag{118}$$

where

$$y_{j,t} = A_t h_{j,t} \tag{119}$$

$$p_{j,t} = \left(\frac{y_{j,t}}{Y_t} \right)^{\frac{1}{-\mu}} P_t. \tag{120}$$

Firm optimization

Substituting in for the demand function and production function yields the following decision problem:

$$\max_{y_{j,t}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{-\gamma}}{C_0^{-\gamma}} \left[\left(\frac{y_{j,t}}{Y_t} \right)^{-\frac{1}{\mu}} P_t y_{j,t} - \frac{W_t}{A_t} y_{j,t} \right] \right\} \tag{121}$$

First order condition:

$$\left(\frac{y_{j,t}}{Y_t} \right)^{-\frac{1}{\mu}} P_t - \frac{1}{\mu} \left(\frac{y_{j,t}}{Y_t} \right)^{-\frac{1}{\mu}-1} P_t \frac{y_{j,t}}{Y_t} = \frac{W_t}{A_t} \tag{122}$$

$$\left(\frac{y_{j,t}}{Y_t} \right)^{-\frac{1}{\mu}} P_t \left[1 - \frac{1}{\mu} \right] = \frac{W_t}{A_t} \tag{123}$$

$$p_{j,t} = \frac{\mu}{\mu - 1} \frac{W_t}{A_t}. \tag{124}$$

Firm optimization II

$$p_{j,t} = \frac{\mu}{\mu - 1} \frac{W_t}{A_t}. \tag{125}$$

- All firms set the same price and charge a mark-up, $\lambda = \frac{\mu}{\mu - 1}$, over marginal costs.
- As a result, they make a profit:

$$\pi_{j,t} = y_{j,t} \left[\frac{\mu}{\mu - 1} \frac{W_t}{A_t} - \frac{W_t}{A_t} \right] = h_{j,t} W_t \frac{1}{\mu - 1}. \tag{126}$$

Deriving aggregates

We are now going to use the firm optimality and aggregate these to derive:

- aggregate prices.
- real wages.
- aggregate profits.
- aggregate output.

Aggregate prices

Combining the aggregate price index,

$$P_t = \left(\int_0^1 p_{j,t}^{1-\mu} dj \right)^{\frac{1}{1-\mu}},$$

with (125) yields

$$P_t = \left(\int_0^1 \left(\lambda \frac{W_t}{A_t} \right)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \tag{127}$$

$$= \lambda \frac{W_t}{A_t}. \tag{128}$$

because for a constant a, we have $\int_0^1 a = a$.

Real wages

This implies for real wages

$$\frac{W_t}{P_t} = \frac{A_t}{\lambda}. \tag{129}$$

Workers are paid their marginal product divided by the mark-up that firms charge.

Aggregate profits

Aggregate profits are:

$$\Pi_t = \int_0^1 \pi_{j,t} dj = H_t W_t \frac{1}{\mu - 1}. \tag{130}$$

Profits are simply a constant fraction of the wage bill.

Aggregate output

Aggregate output is given by:

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} dj \right)^{\frac{\mu}{\mu-1}}. \tag{131}$$

As all intermediate good producers charge the same price, they have the same output, and we have

$$Y_t = \int_0^1 y_{j,t} dj = \int_0^1 A_t h_{j,t} dj = A_t H_t. \tag{132}$$

Household budget constraint

Finally, we can simplify the household's budget constraint. In equilibrium, bonds need to be in zero net supply: $B_t = B_{t+1} = 0$.

$$P_t C_t = W_t H_t + \Pi_t = W_t H_t \lambda. \tag{133}$$

Substituting for the real wage gives us

$$C_t = \frac{A_t}{\lambda} H_t \lambda = Y_t, \tag{134}$$

i.e., all output is consumed.

Solution

So far, we have the following set of equations:

$$C_t^{-\gamma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\gamma} \frac{1 + i_t}{1 + \pi_{t+1}} \right\} \tag{135}$$

$$\phi H_t^{\eta} = C_t^{-\gamma} \frac{W_t}{P_t} \tag{136}$$

$$\frac{W_t}{P_t} = \frac{A_t}{\lambda} \tag{137}$$

$$C_t = Y_t \tag{138}$$

$$Y_t = A_t H_t \tag{139}$$

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1}. \tag{140}$$

We cannot pin down the expected inflation rate AND the interest rate.

Solution II

To close the model, we simply assume an exogenous process for the nominal interest rate. It fluctuates around its steady state (assuming $\pi^{ss} = 0$) but reacts to changes in inflation. Later in the course, we will see that such an interest rate rule is linked to central banks behavior:

$$i_t = \frac{1}{\beta} - 1 + \kappa_\pi \pi_t + \epsilon^i_t. \tag{141}$$

Moreover, we introduce shocks to mark-ups. Those follow an AR(1) process in logs around their log steady state:

$$\ln \lambda_{t+1} = (1 - \rho_\lambda) \ln \left(\frac{\mu}{\mu - 1} \right) + \rho_\lambda \ln \lambda_t + \epsilon^\lambda_{t+1} \tag{142}$$

Calibration

- We are going to use the same calibration as before with an inverse labor supply elasticity $\eta = 0.5$ This leads to $\phi = 13$ to match $H^{ss} = 0.33$.
- I set $\kappa_\pi = 1.1$, more on that later in the course.
- Changes in mark-ups lead to changes in real wages and, hence, hours. $\rho_\lambda = 0.5$ matches the autoccorelation of hours.
- Finally, I use σ_i to match the standard deviation of inflation and set $\sigma_\lambda = 0.05$ which is arbitrary.

A productivity shock

We can see directly that real wages increase:

$$\frac{W_t}{P_t} = \frac{A_t}{\lambda_t}. \tag{143}$$

Combine the budget constraint, (134), with the optimal hours decisions, (105), to get

$$\phi H_t^{\eta+\gamma} = \frac{A_t^{1-\gamma}}{\lambda_t} \tag{144}$$

$$H_t = \left(\frac{1}{\phi\lambda_t}\right)^{\frac{1}{\eta+\gamma}} A_t^{\frac{1-\gamma}{\eta+\gamma}}. \tag{145}$$

In a model without capital, households have no reason to work harder after a positive productivity shock to build up capital. Hence, resulting from a wealth effect, hours decrease after an increase in productivity. Hours are constant with log utility because the wealth and substitution effect cancel.

A productivity shock II

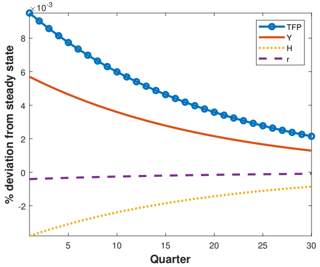
Substituting the optimal hours into the production function yields

$$Y_t = \left(\frac{1}{\phi\lambda_t}\right)^{\frac{1}{\eta+\gamma}} A_t^{\frac{1+\eta}{\eta+\gamma}} \tag{146}$$

Output increases today. Hence, consumption (output) today is higher than in the future. Hence, the bond equation tells us that the real interest rate, $i_t - \pi_t$, must fall:

$$C_t^{-\gamma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\gamma} \frac{1+i_t}{1+\pi_{t+1}} \right\}. \tag{147}$$

A productivity shock III



- This implies that hours may be countercyclical.
- It implies the real interest rate may be countercyclical.

A mark-up shock

We can see directly that real wages will decrease:

$$\frac{W_t}{P_t} = \frac{A_t}{\lambda}.$$

This reduces the incentives to work so hours decline:

$$H_t = \left(\frac{1}{\phi\lambda_t}\right)^{\frac{1}{\eta+\gamma}} A_t^{\frac{1-\gamma}{\eta+\gamma}}.$$

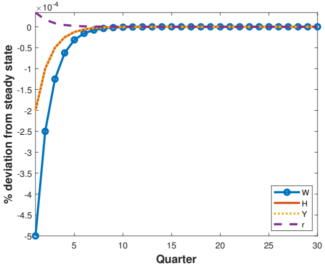
A decrease in hours will decrease output and consumption by the same amount:

$$Y_t = Y_t = \left(\frac{1}{\phi\lambda_t}\right)^{\frac{1}{\eta+\gamma}} A_t^{\frac{1+\eta}{\eta+\gamma}} = C_t.$$

As consumption growth will be positive, the real interest rate needs to increase:

$$C_t^{-\gamma} = \beta\mathbb{E}_t\left\{C_{t+1}^{-\gamma}\frac{1+i_t}{1+\pi_{t+1}}\right\}.$$

A mark-up shock II



- This implies that hours may be procyclical. Its cyclicality will depend on the relative importance of productivity and mark-up shocks.
- It implies the real interest rate will be countercyclical.

Comparing models and data

	Y	C	H	TFP	w	r
			Data			
Std. %	1.61	1.25	1.9	1.25	0.96	1.02
			$\sigma_i = \sigma_l = 0$			
Std. %	1.56	0.45	0.52	1.24	1.10	0.06
			$\sigma_i = 0.004; \sigma_l = 0.0005$			
Std. %	0.74	0.74	0.5	1.24	1.24	0.35

Correlations

	Y	C	H	TFP	w	r
			Data			
Y	1					
C	0.78	1				
H	0.87	0.68	1			
TFP	0.79	0.71	0.49	1		
w	0.12	0.29	-0.06	0.34	1	
r	0.24	0.12	0.40	0.05	-0.13	1
			$\sigma_i = 0.004; \sigma_l = 0.0005$			
Y	1					
C	1	1				
H	-1	-1	1			
TFP	1	1	-1	1		
w	1	1	-1	1	1	
r	-0.15	-0.15	0.15	-0.15	-0.15	1

Discussion

Improvements:

- Consumption and the real interest rate are more volatile.
- The real interest rate is no longer strongly procyclical and much more volatile.

Deterioration:

- Output is not volatile enough.
- Without capital, consumption is as cyclical as output.
- Without capital, hours are countercyclical.